ME 323: FLUID MECHANICS-II

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Lecture-06

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Operation of Converging-Diverging Nozzle

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Choking Phenomena cont...

Recap

The **maximum possible mass flow rate through a nozzle is;**

$$\dot{m}_{\max} = \rho^* A^* V^*$$

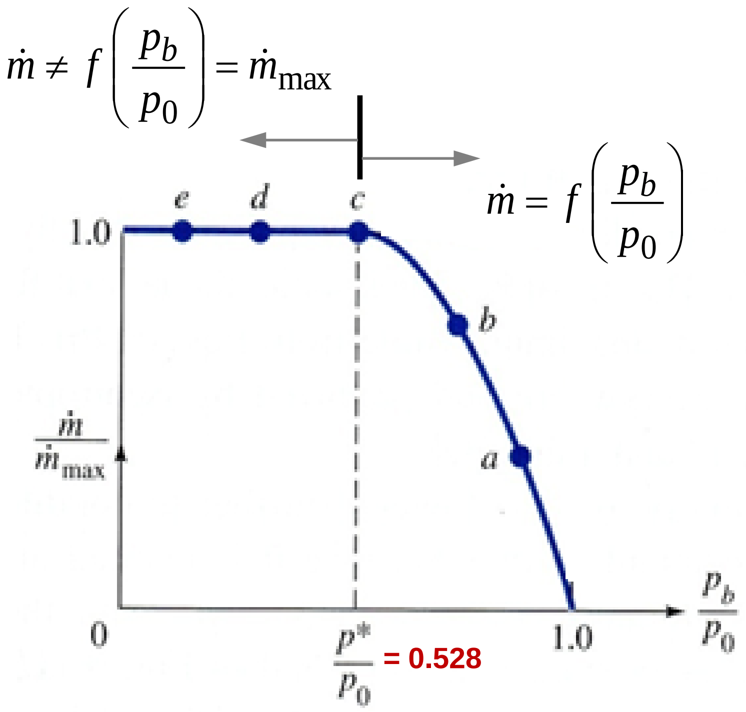
$$\Rightarrow \dot{m}_{\max} = \rho_0 \left(\frac{2}{k+1}\right)^{1/(k-1)} A^* \left(\frac{2k}{k+1} R T_0\right)^{1/2}$$

$$\Rightarrow \dot{m}_{\max} = \frac{p_0}{RT_0} \left(\frac{2}{k+1}\right)^{1/(k-1)} A^* \left(\frac{2k}{k+1} R T_0\right)^{1/2}$$

$$\Rightarrow \dot{m}_{\max} = k^{1/2} \left(\frac{2}{k+1}\right)^{\frac{k+1}{2(k-1)}} A^* \frac{p_0}{\sqrt{RT_0}}$$

$$\because V^* = M^* a^* = (1.0)\sqrt{kRT^*}$$

$$\Rightarrow V^* = \sqrt{\frac{2k}{k+1} R T_0} \quad ; \quad \frac{T^*}{T_0} = \frac{2}{k+1}$$



For air; *k* = 1.4 and *R* = 287 J/kgK:

$$\boxed{\dot{m}_{\max} \approx 0.04 \frac{p_0 A^*}{\sqrt{T_0}} \qquad (\text{kg/s})}$$

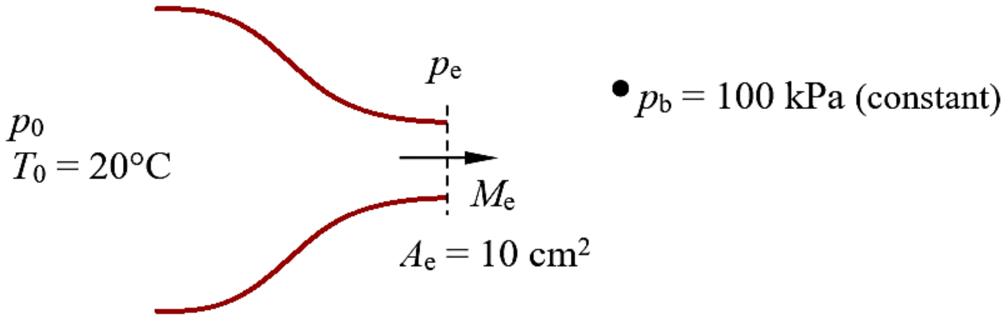
For isentropic flow through a duct;

the maximum mass flow possible is

- **proportional to the throat area, A***
- **proportional to stagnation pressure, p_0** and
- **inversely proportional to the square root of the stagnation temperature, T_0.**

Problem

Air is being discharged to atmosphere (p_b = 100 kPa) through a converging nozzle as shown in figure. The air is being feed from a large reservoir in which the pressure is continuously increased from 200 kPa to 600 kPa. During this operation, the reservoir temperature is maintained constant at 20°C.



(a) Complete the table:

p_0 (kPa)	200	300	400	500	600
p_e (kPa)					
M_e					
$\dot{m}$ (kg/s)					

(b) Plot $\dot{m}$ vs. p_0

(c) Is the nozzle choked or not? Justify your comment.

Supersonic Nozzle Operation

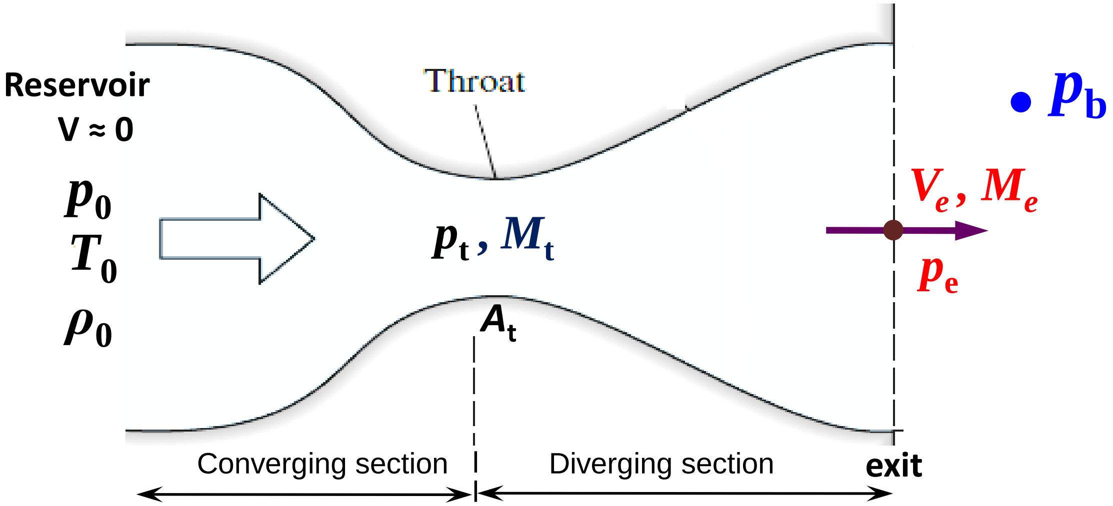
Consider a converging-diverging (C-D) nozzle in which a gas is flowing from a large reservoir (V ≈ 0). Thus, the reservoir is at stagnant condition. The reservoir pressure is p_0 which is **kept constant** throughout the operation (steady flow).

The **flow structure** inside and outside the nozzle is dependent on the magnitude of available **back pressure, p_b** (where jet will be exhausted).

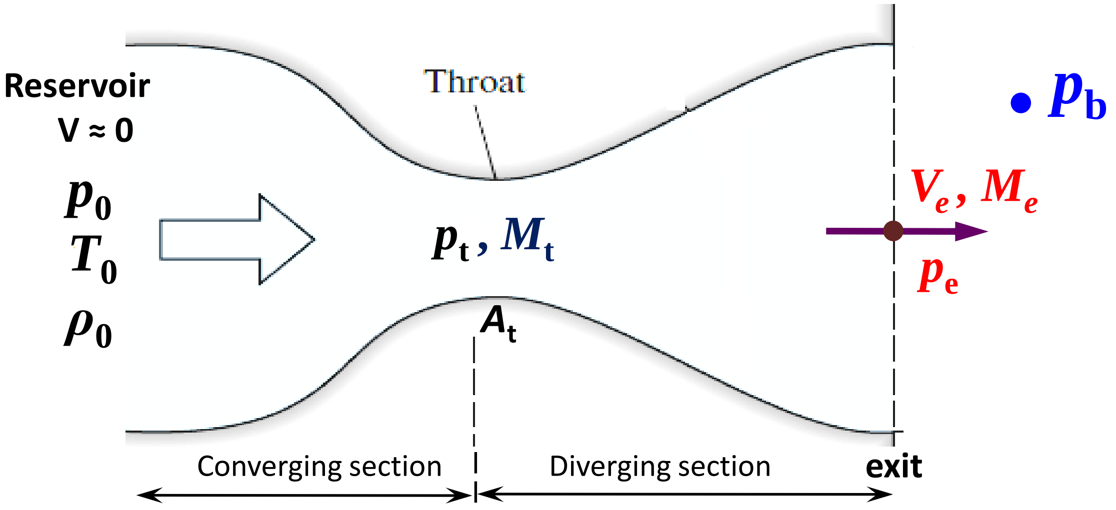
Now, the back pressure, p_b **will be decreasing** in a well controlled manner while p_0 **is remained fixed**.

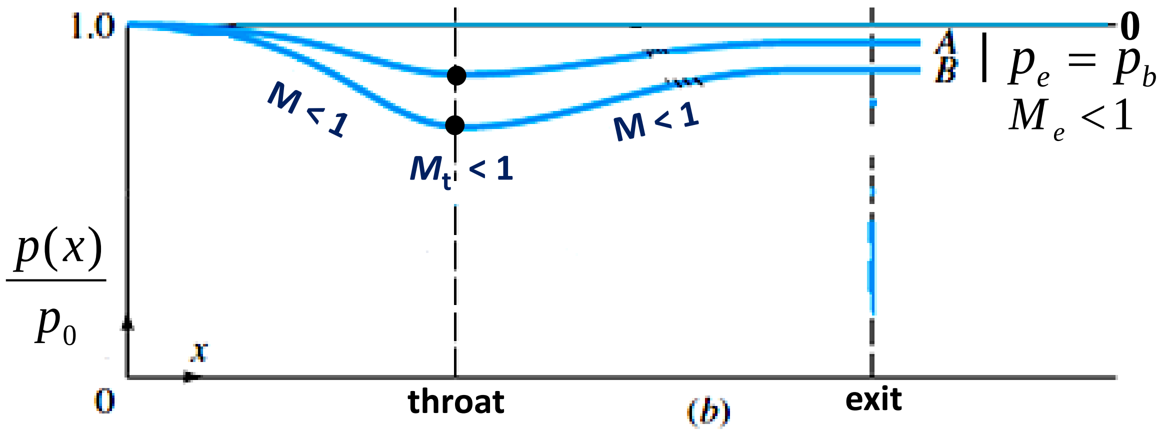
The flow condition is defined by the parameter:

$$\frac{p_b}{p_0} \quad \text{or} \quad \frac{p_0}{p_b} \ (\text{NPR})$$



Supersonic Nozzle Operation





Case - 0 : There will be no flow when $p_b = p_0$ i.e.

$$\frac{p_b}{p_0} = 1.0 \qquad \dot{m}\big|_0 = 0$$

Cases-A and **B** ($p_b < p_0$) represent the **subsonic isentropic flow** inside the nozzle. In these cases, Mach number will increase in the converging section and will reach maximum at the throat **but at subsonic value (*M* < 1)** and then decreases in the diverging portion **(subsonic operation)**.

The exit jet Mach number, M_e will be less than 1. **(Venturimeter operation !!)**

$$\frac{p_b}{p_0} < 1.0 \qquad \dot{m}\big|_B > \dot{m}\big|_A > \dot{m}\big|_0 = 0$$

For example:

$$A: \frac{p_b}{p_0} = 0.9 \quad \therefore M_e \approx 0.4$$

$$B: \frac{p_b}{p_0} = 0.8 \quad \therefore M_e \approx 0.57$$

Supersonic Nozzle Operation

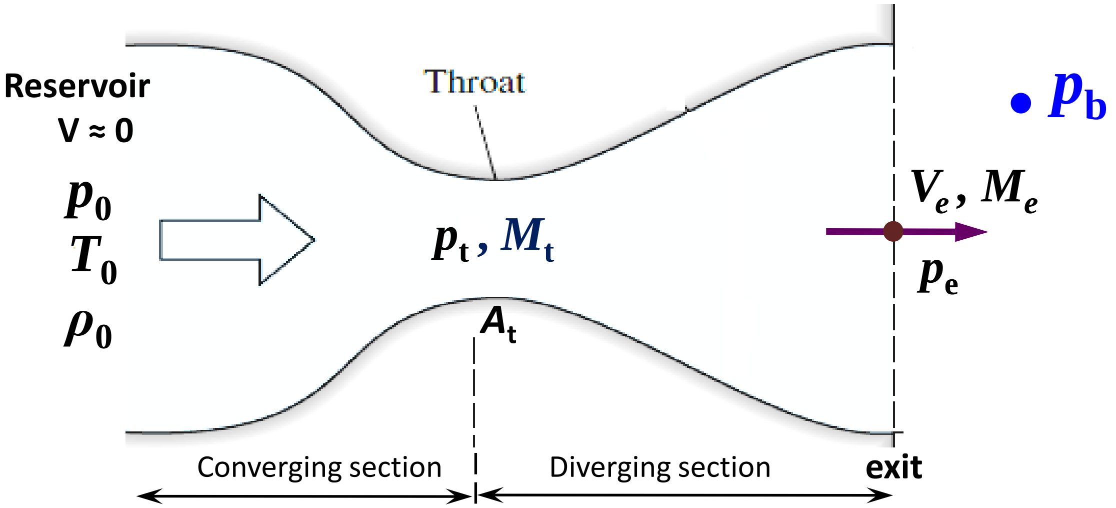
As the back pressure is decreased, more flow is further induced through the nozzle until eventually **sonic flow occurs** at the throat. This is **Case-C.**

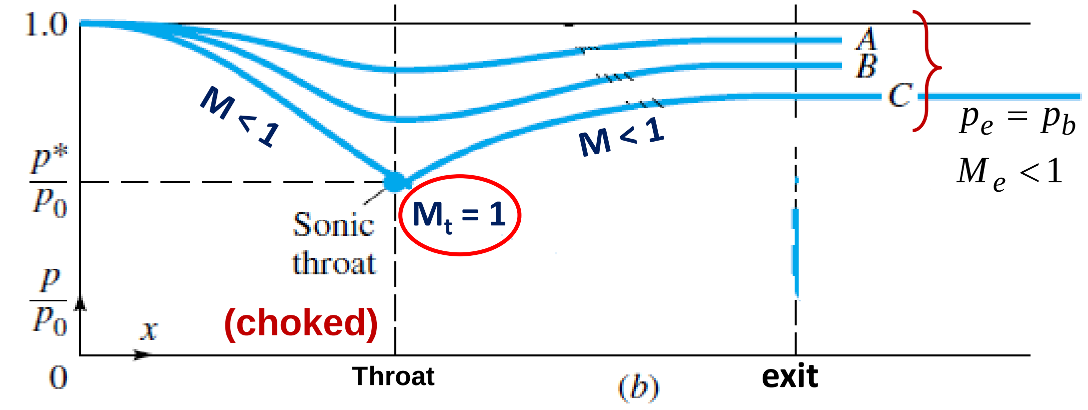
At **Case-C (M_t = 1.0, p_t = 0.528p_0)**, the throat is **just reached** at critical condition with M_t = 1.0. However, after the throat, the flow could not expand further, and the flow is still subsonic in the diverging portion of the nozzle (M_e<1). ***Nozzle is chocked*** at **case C**, and **the mass flow rate is the maximum.**

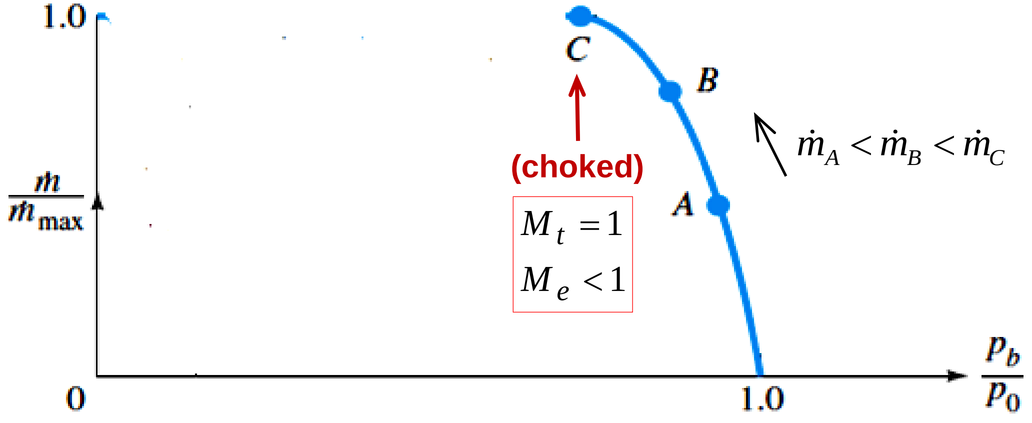
Further decrease of back pressure **CANNOT** be "transmitted/sensed" upstream of the throat; thus, for all the back pressures below that of Case-C, the reservoir continues to deliver the same flow rate as in Case-C and **NO** additional mass will be induced **(choking phenomena).**

The pressure distribution up to the throat will remain the same for the next other cases.

Is this the design operation of nozzle?? (No)



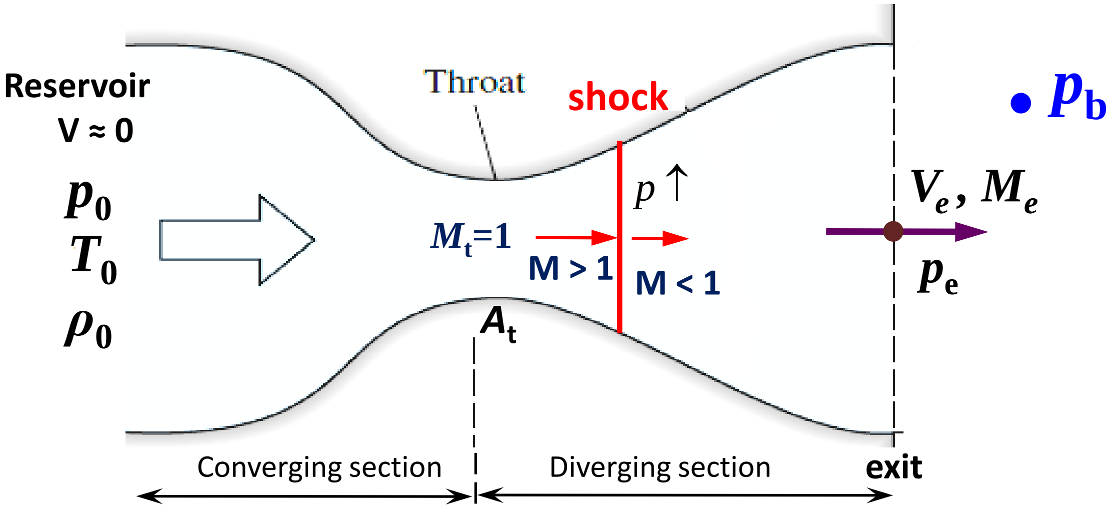




Supersonic Nozzle Operation

If back pressure is decreased from the magnitude as in **Case-C**, the flow could be able to expand in the diverging section.

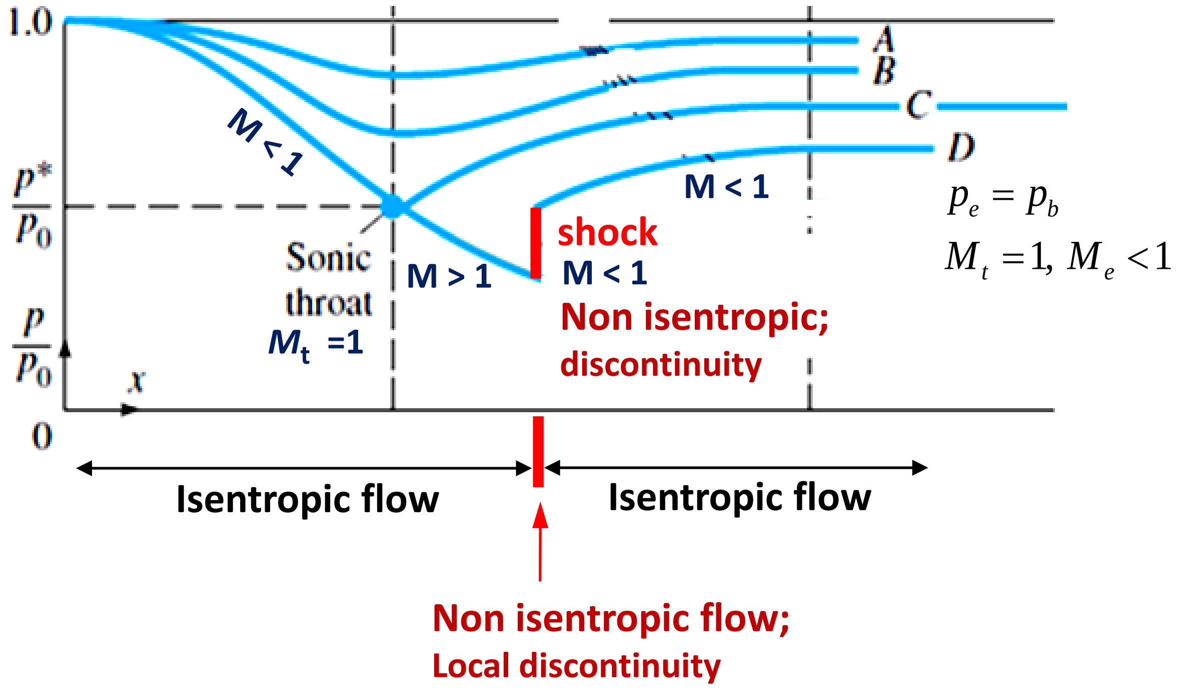
In this case, **Case-D**; some portion of the diverging section will be **supersonic (M>1)**. This portion will be terminated by a **normal shock wave (sudden jump in pressure)** and the downstream section will be **subsonic** again **(M_e<1).**



The shock wave is assumed to be a local discontinuity across which the flow properties suddenly change. Shock wave decelerates the flow.

NO isentropic solution is allowed in the full diverging section of the nozzle. However, *flow from converging section to the location just before the shock wave is isentropic.*

Similarly, flow from the location just downstream of the shock to the exit of the nozzle is isentropic.

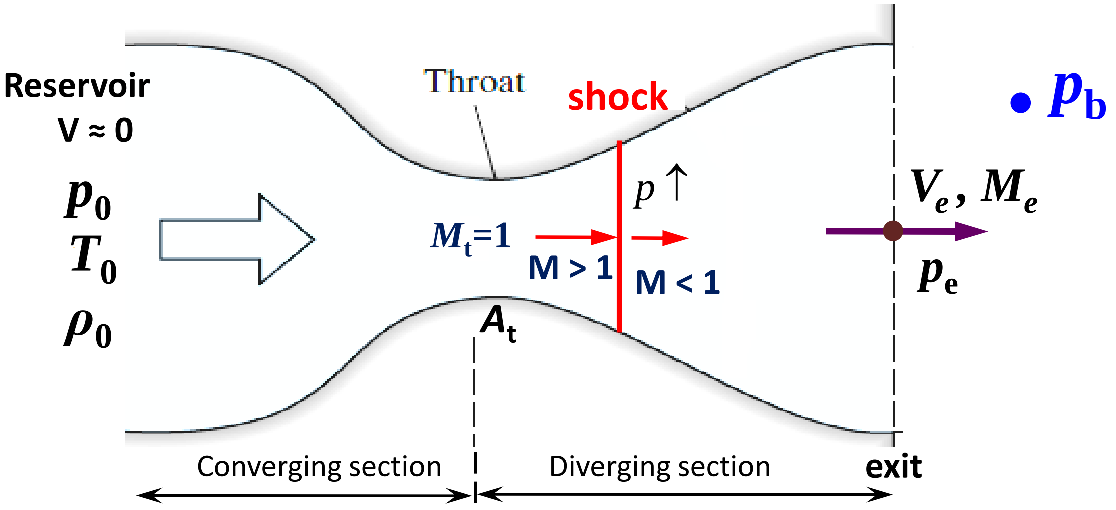


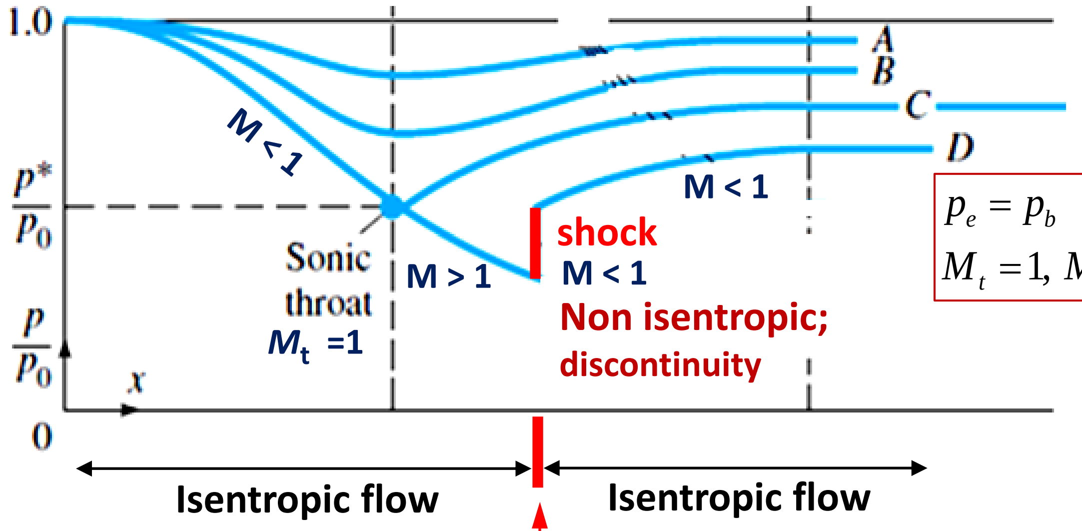
Supersonic Nozzle Operation

Shock properties need to be calculated using **shock relations** (to be discussed in next classes).

The location of the normal shock wave is determined by the requirement that the increase of static pressure across the shock wave (local discontinuity) **plus** that in the diverging portion of the subsonic flow behind the shock be (due to isentropic expansion) just right to achieve the exit pressure, p_e equal to p_b.

Mass flow rate will remain the same (and maximum) in this case as found in **Case-C (Choked).**





$p_e = p_b$
$M_t = 1, M_e < 1$

Non isentropic flow;
Local discontinuity

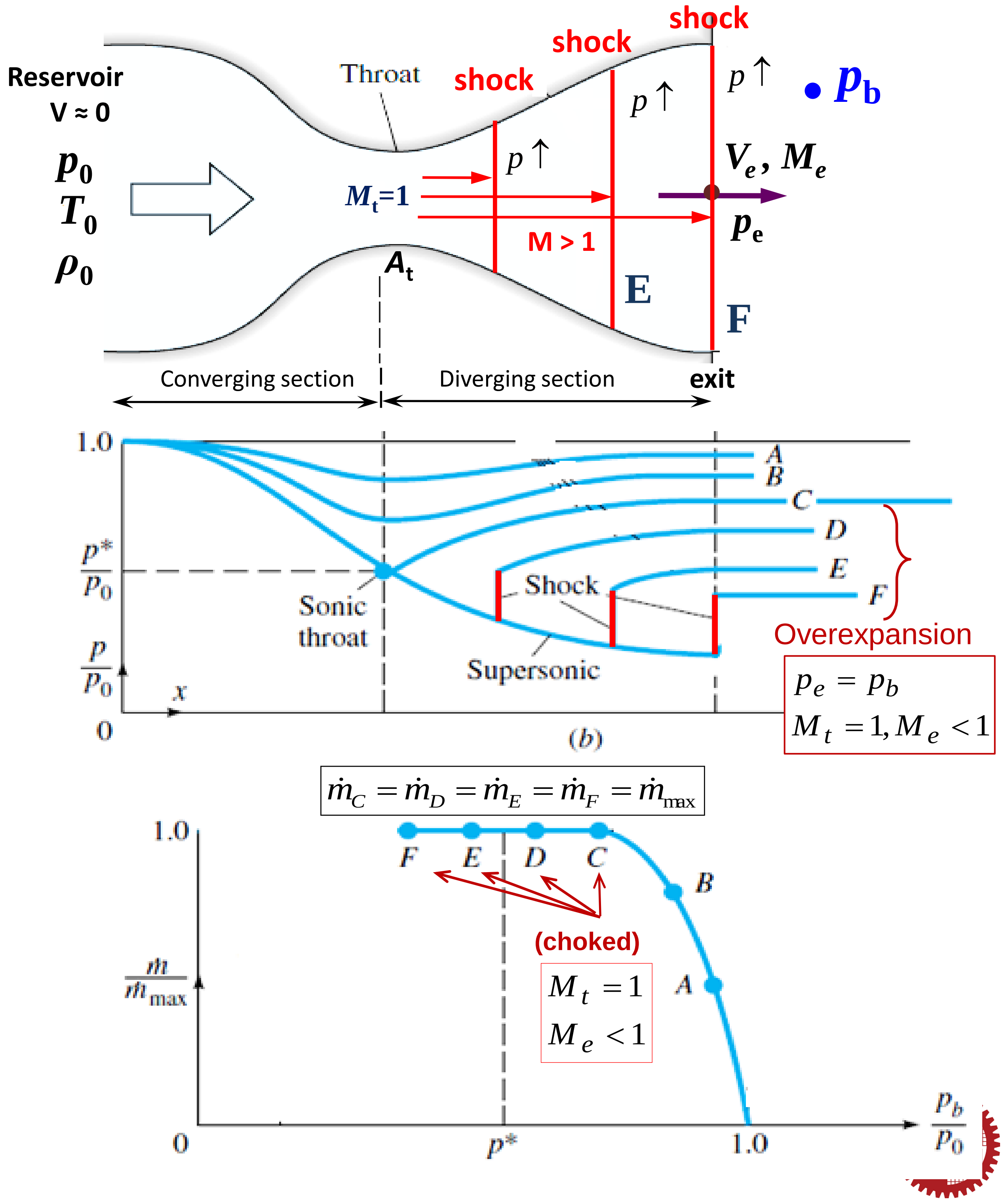
Supersonic Nozzle Operation

As the back pressure is reduced further, the normal shock wave will move downstream, closer to the nozzle exit (**Cases-E** and **F**).

These cases are known as **“Overexpansion”** since the nozzle is **overly expanded inside the nozzle** than the available back pressure in the discharge side. **(at low altitudes)**

Mass flow rate remains the constant which is maximum for a given nozzle throat size and reservoir conditions.

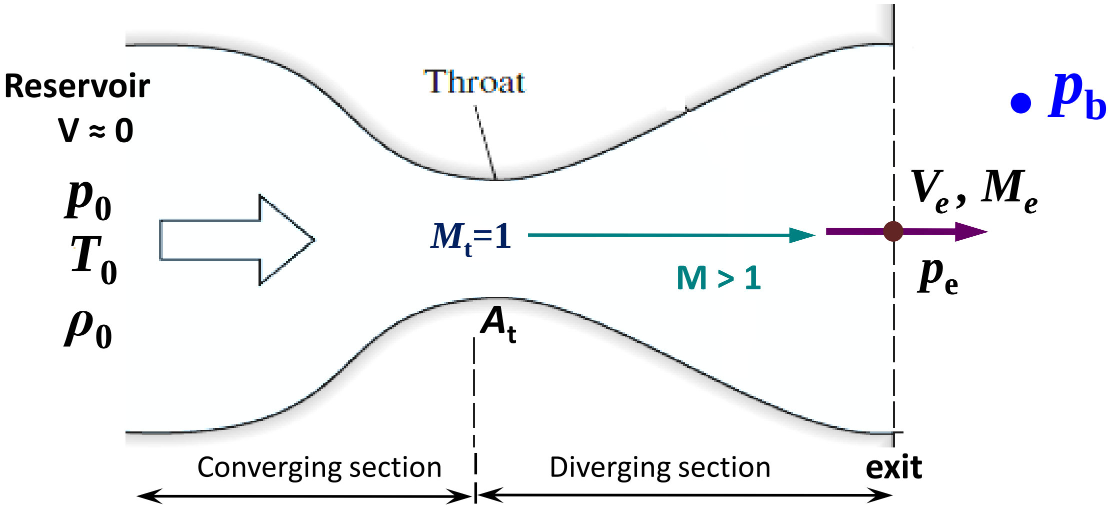
Overexpansion is an Off-design operation of nozzle.



Supersonic Nozzle Operation

When an expansion (supersonic flow, M>1) in the complete diverging section is possible (**Case-H**), then the case is known as **"ideal expansion"** (or correct expansion). Also known as **design operation (M_e >1)**.

In this case, the nozzle exit pressure is exactly the same as the back pressure (p_e=p_b). The **flow is isentropic throughout the nozzle.**

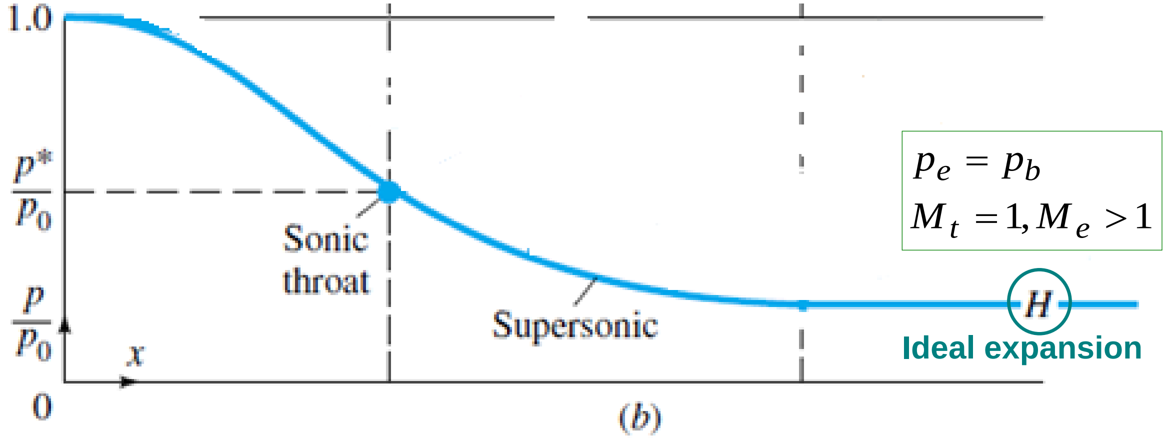


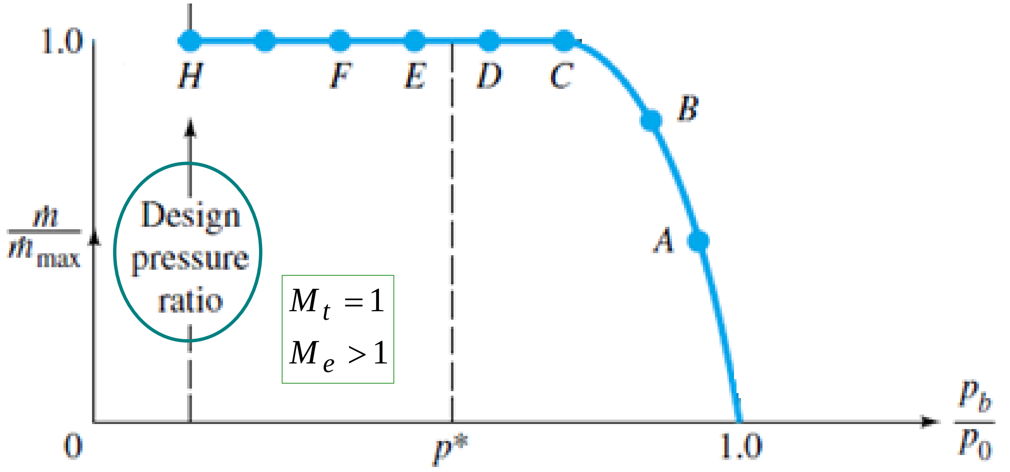
Design operating condition (i.e. the design back pressure) can be calculate knowing the **expansion ratio, *ε*** at nozzle exit.

First, the design Mach number can be calculated using area-Mach relation and this Mach number can be used to determine the required static pressure (p_b) using isentropic relation.

$$\frac{A_e}{A^*}=\frac{1}{M_e}\frac{\left(1+0.2M_e^2\right)^3}{1.728}\rightarrow M_e=?\ (>1.0)$$

$$\frac{p_0}{p_b}=[1+0.2M_e^2]^{3.5}\rightarrow p_b=?$$



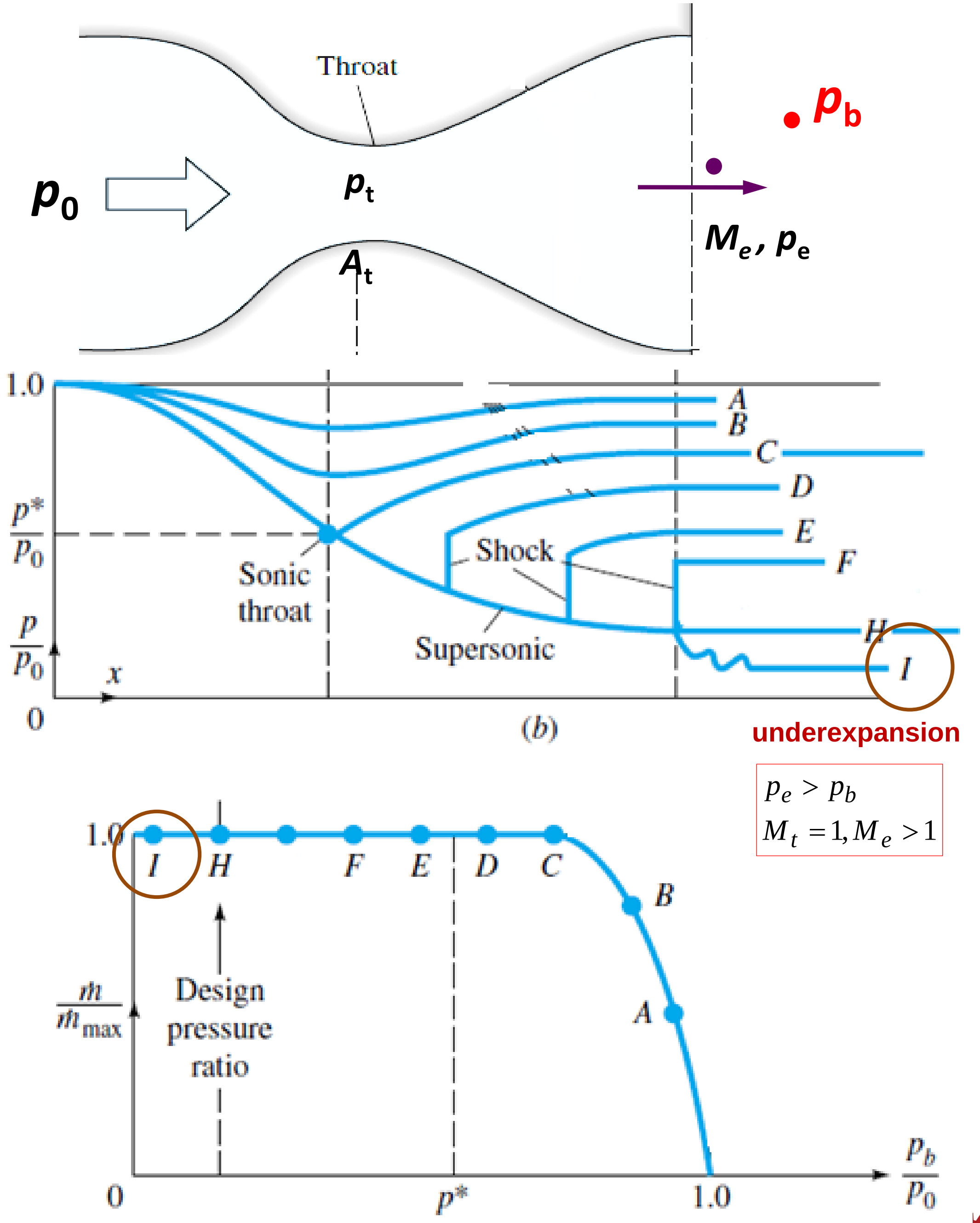


Supersonic Nozzle Operation

Case-I:
However, when the back pressure is reduced further (below Case-H), then the case is known as **“underexpansion”**. This case is seen when nozzle is operating at **higher altitudes (several km from the sea-level)**.

The exit pressure is higher than the back pressure $(p_e)_{\text{case-I}} > p_b$, and hence the flow is capable of additional expansion after leaving the nozzle. This is accompanied by generation of **expansion waves and shock waves** from the nozzle exit.

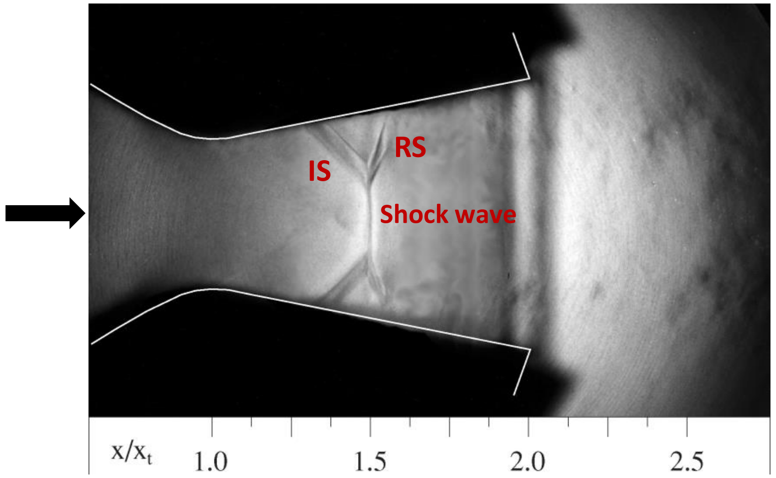
The flow is isentropic inside the nozzle, however, the flow is **NOT isentropic outside the nozzle**.

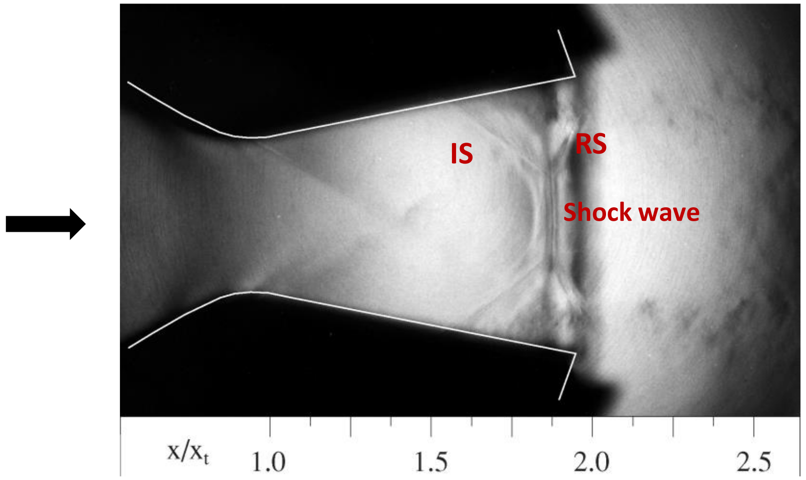


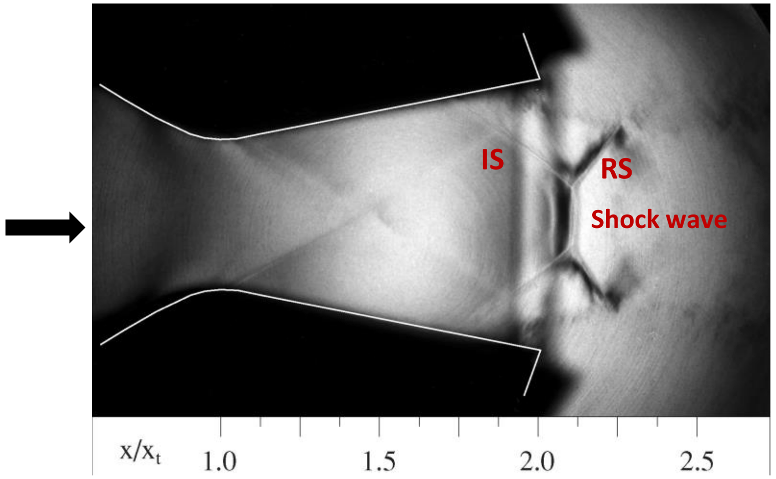
$p_e > p_b$
$M_t = 1, M_e > 1$

Supersonic Nozzle Operation

Overexpansion



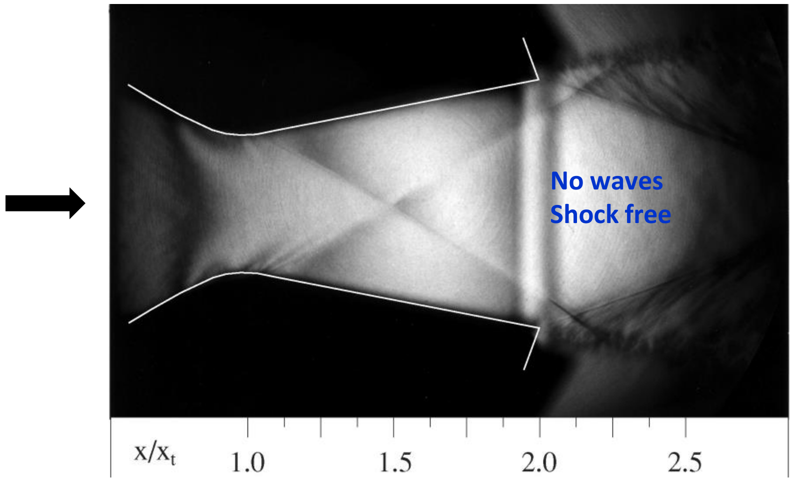




IS: Incident shock wave
RS: Reflected shock wave

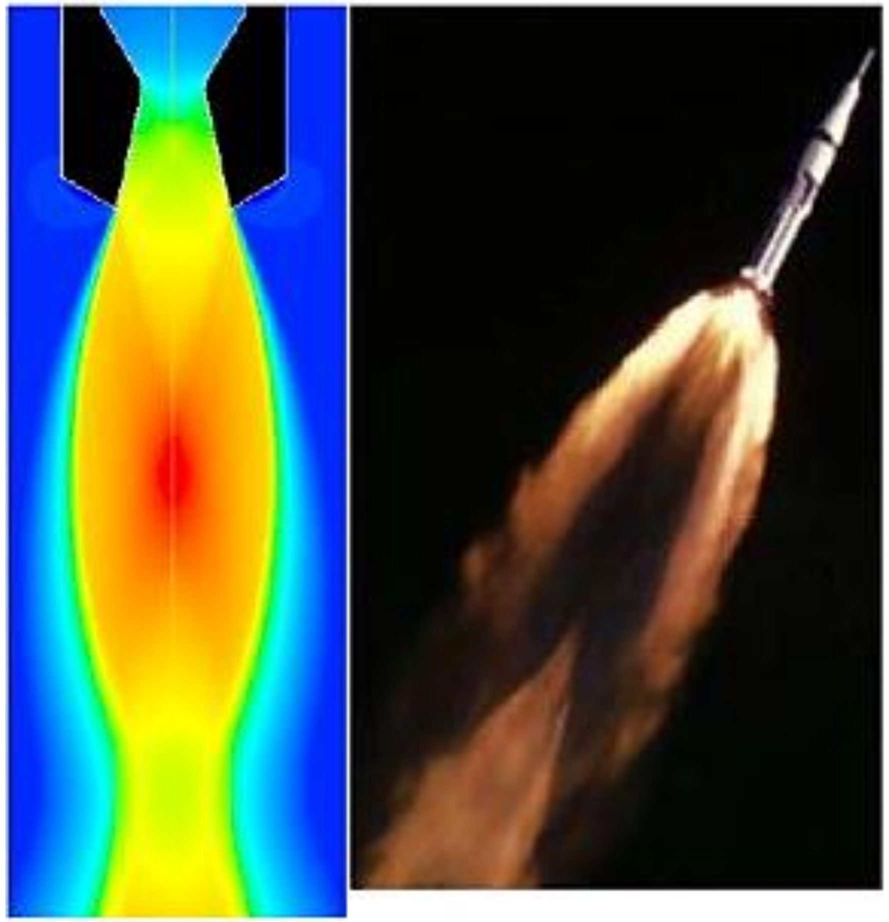
Source: Hunter, Journal of Propulsion and Power, 20 (3), 2004. pp. 527-532

Ideal Expansion



Source: Hunter, Journal of Propulsion and Power, 20 (3), 2004. pp. 527-532

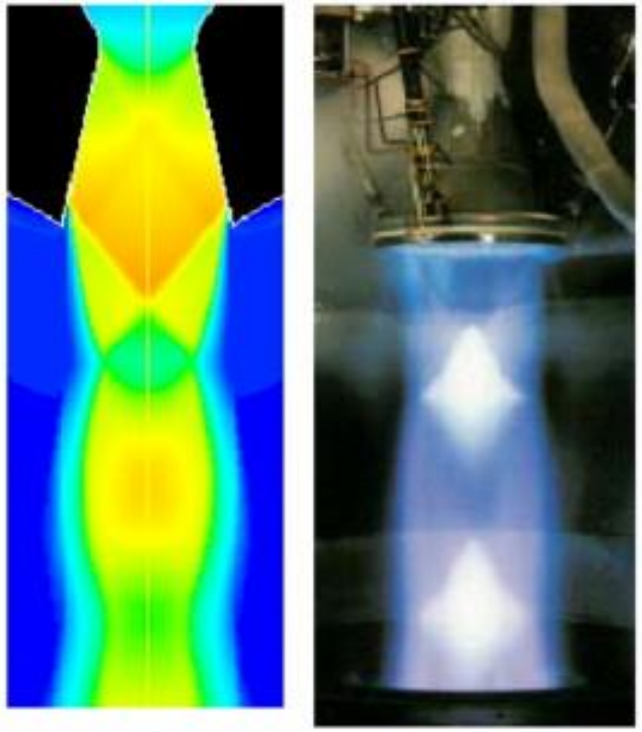
Underexpanded jet



Present Prediction
NPR=12

photographed during launch of
Saturn 1-B,
Östlund [41]

b- Under-expanded flow condition



Present Prediction
NPR =2.4

Östlund [41]

a- Over-expanded flow condition